Auction Quotas with a Foreign Duopoly

Ling Hui Tan

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Prepared by Ling Hui Tan¹

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Abstract

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This paper uses a partial equilibrium framework to compare the welfare consequences of different methods of quota administration relative to free trade under imperfect competition. It shows that a country importing a good from foreign duopolists may improve its welfare by setting a quota at the free trade quantity and giving a fraction of the quota licenses to the duopolists while auctioning off the rest.

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Author's E-Mail Address: ltan@imf.org

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	Contents	Page
I.	Introduction	3
Π.	The Model	4
III	. Optimal Policy	7
IV	Conclusion	10
1.	ext Figures: (V, λ) Combinations Resulting in a Positive License Price	11
Re	eferences	21
Αŗ	opendix I: Derivations of Case One and Case Two	13
	opendix Figures:	
A.	1. Firm 1's Profit Functions when $P_2 > \overline{P_2}$	15
	2. Firm 1's Profit Functions when $P_2 < \hat{P}_2$	
A.	3. Firm 1's Profit Functions when $\hat{P}_2 < P_2 < \overline{P}_2$	17
A.	5. Firm 1's Profit Functions when $\overline{P}_2 < P_2 < \hat{P}_2$	19
A.	6. Case Two: VER at the Free Trade Quantity	20

I. INTRODUCTION

Quantitative restrictions have been an instrument of trade policy in industrial countries in the last two decades.² For example, in the United States and Europe, such restrictions have been imposed on products ranging from textiles and apparel to steel and automobiles.

There are different ways to implement quantitative restrictions which basically involve the allocation of import rights. One example is the voluntary export restraint, or VER, which is a bilateral agreement between the importing and exporting country whereby the latter "voluntarily" limits its exports to some previously negotiated amount. This is tantamount to giving all the import rights, and consequently all the quota rents, to the foreign suppliers.

Another method is to auction off the import licenses. This is often recommended as a means by which the government of the importing country can retrieve the quota rents. Auction quotas have been implemented in Australia and New Zealand and they have been widely debated in the United States since their authorization in 1979.³ The efficacy of such a policy, however, depends crucially on the structure of the product market. Auction quotas will raise license revenue for the importing country when there is perfect competition in the product market, but they may be less effective in doing so when there is imperfect competition. This is because the foreign producers will take the license price into consideration when they make their pricing decision. Unless the quota is very restrictive, they will simply raise the product price up to the demand price for the good, effectively stripping the licenses of any value.⁴

While practical policy and the theoretical literature have focused on these two forms of quota implementation, Krishna (1991) considers an interesting alternative, whereby the importing country gives some of the licenses to the foreigners and auctions off the rest. The idea is that the (imperfectly-competitive) producers now derive revenue both from their sales and from the licenses they hold; therefore under certain conditions, they will have an incentive to maintain a positive value for the licenses. She shows that when a country imports from a foreign monopolist who also sells to the foreign market and is unable to price discriminate, a Pareto-improving policy could be for the country to set a quota at its free trade level of imports, give away a fraction of the quota licenses to the monopolist, and auction the remainder.

² Quantitative restrictions in the form of straightforward import quotas have traditionally been—and in most cases, continue to be—widely used in many developing countries.

³ See Bergsten, *et. al* (1987).

⁴ See Krishna (1988).

This paper extends Krishna's analysis to the case where the foreign suppliers are Bertrand duopolists. Unlike Krishna (1991), this paper assumes that the home market is segmented from the foreign market (thereby allowing for international price discrimination) so as to focus on the price competition between the two foreign producers in the domestic market. Using an example with linear demands, it is shown that the optimal fraction of licenses to give away depends on the degree of substitutability between the products of the foreign duopolists.

II. THE MODEL

Consider two symmetric foreign firms selling a differentiated product solely to a third market, the home country. For simplicity, assume there is no foreign market and no home production of the restricted good. Firm *i* faces a linear demand for its product given by:

$$Q_i(P_i, P_j) = A - kP_i + P_j,$$
 $i, j = 1, 2 \quad i \neq j$ (1)

where k > 1, so the products are imperfect substitutes. The two firms compete in prices.

Under free trade, each firm will maximize its profit, taking as given the price charged by its rival. Abstracting from costs, therefore, Firm i will maximize $P_i(A-kP_i+P_j)$, given P_j , and set:

$$B_i(P_j) = \frac{A + P_j}{2k}. (2)$$

Equation (2) defines Firm i's free trade best response function.

In equilibrium, each firm will sell $Q^F = Ak/(2k-1)$. The free trade welfare level, W^F , is simply the consumer surplus associated with the free trade quantity, $V^F (= 2Q^F)$.

Now suppose the home government imposes a quota of V units on both firms combined, where $V \leq V^F$, and gives a fraction, λ , of the quota licenses to the foreign producers. Suppose the two firms share the licenses equally so each firm is awarded $\lambda V/2$ licenses.

⁵ Under Bertrand competition, price is the strategic variable: each firm assumes that its rival(s) will choose a price and adjust output to meet demand at that price, hence, each firm sets its price to maximize profit, taking as given the price charged by its rival(s).

⁶ Although Krishna (1993) analyzes the outcome of an auction quota with a foreign duopoly, she only considers the case where the entire quota is auctioned. This paper introduces a second policy parameter besides the quota level, namely, the share of licenses awarded to the foreign duopolists.

The sequence of events is such that the home authorities first announce V and λ , then the firms choose their prices, and finally, the license market clears.

The model is solved backward, beginning with the license market clearing condition:

$$Q_1(P_1 + L, P_2 + L) + Q_2(P_1 + L, P_2 + L) = V.$$
(3)

The equilibrium license price is given by:

$$L(P_1, P_2; V) = \begin{cases} \frac{2A - V - (k-1)(P_1 + P_2)}{2(k-1)} & \text{if } P_1 + P_2 < \frac{2A - V}{k-1} \\ 0 & \text{if } P_1 + P_2 \ge \frac{2A - V}{k-1}. \end{cases}$$
(4)

The license price is positive only if the prices charged by the two firms result in excess demand for the good at the prevailing quota level, i.e., if $Q_1 + Q_2 > V$; if $Q_1 + Q_2 \leq V$, the license price will be zero.

Using (4), Firm i's perceived demand can be written as:

$$Q_i(P_i + L, P_j + L) = A - kP_i + P_j - (k - 1)L.$$
(5)

Each firm will maximize its profit, taking as given its rival's price, the quota level, and the number of licenses it is awarded. Note that the profit function also includes the value of the licenses held by the firm. Hence, Firm *i*'s profit function is $P_iQ_i(P_i+L,P_j+L)+\lambda VL/2$, which can be written as:

$$\widetilde{\pi}_{i}(P_{i}; P_{j}, V) = P_{i}(A - kP_{i} + P_{j}) + \left(\frac{\lambda V}{2} - (k - 1)P_{i}\right) \left(\frac{2A - V - (k - 1)(P_{i} + P_{j})}{2(k - 1)}\right)$$
(6)

when the license price is positive, and:

$$\pi_i(P_i; P_j) = P_i(A - kP_i + P_i) \tag{7}$$

when the license price is zero. This gives rise to a best response function that is kinked, as derived below.

From (4), the combinations of P_1 and P_2 which result in a zero equilibrium license price are given by:

$$P_1 + P_2 = \frac{2A - V}{k - 1}. (8)$$

Call this the $L(\cdot) = 0$ line. The best response function will be kinked along this line.

When the license price is positive, i.e., for combinations of P_1 and P_2 below the $L(\cdot) = 0$ line, the first order condition from maximizing $\tilde{\pi}_i$ yields the following best response function for Firm i:

$$\widetilde{B}_i(P_j; V, \lambda) = \frac{P_j}{2} + \frac{V(2 - \lambda)}{4(k+1)}.$$
(9)

When the license price is zero, i.e., for combinations of P_1 and P_2 above the $L(\cdot) = 0$ line, Firm i's best response function is just its free trade best response function $B_i(P_j)$ given by Equation (2).

Along the $L(\cdot) = 0$ line, $\partial \widetilde{\pi}_i / \partial P_i = 0$ at:

$$\hat{P}_{j}(V,\lambda) = \frac{2}{3} \left(\frac{2A - V}{k - 1} \right) - \frac{1}{3} \left(\frac{V(2 - \lambda)}{2(k + 1)} \right) \tag{10}$$

and $\partial \pi_i / \partial P_i = 0$ at:

$$\overline{P}_{j}(V) = \left(\frac{2k}{1+2k}\right) \left(\frac{2A-V}{k-1}\right) - \frac{A}{1+2k}.$$
(11)

Firm i's best response from maximizing profit plus license revenue will result in an aggregate demand for the good exactly equal to the quota level and a license price of zero if Firm j sets its price at $\hat{P}_j(V,\lambda)$. Firm i's best response from maximizing profit only will result in a license price of zero if Firm j sets its price at $\overline{P}_j(V)$.

Let Case One be the situation where $\hat{P}_j(V,\lambda) < \overline{P}_j(V)$. From Equations (10) and (11), this is the case for combinations of V and λ such that $V[2+\lambda(1+2k)] < 2A(k+1)$. Using reasoning similar to that found in Krishna (1993), it can be shown that Firm i will charge $P_i(P_j;V,\lambda)$ such that:

$$P_{i}(P_{j};V,\lambda) = \begin{cases} \frac{P_{j}}{2} + \frac{V(2-\lambda)}{4(k+1)} & \text{if } P_{j} \leq \hat{P}_{j}(V,\lambda) \\ \frac{2A-V}{k-1} - P_{j} & \text{if } \hat{P}_{j}(V,\lambda) \leq P_{j} \leq \overline{P}_{j}(V) \\ \frac{A+P_{j}}{2k} & \text{if } P_{j} \geq \overline{P}_{j}(V) \end{cases}$$
(12)

Hence, Firm i will price along $\widetilde{B}_i(P_j;V,\lambda)$ when $P_j < \widehat{P}_j(V,\lambda)$, along $B_i(P_j)$ when $P_j > \overline{P}_j(V)$, and along $L(\cdot) = 0$ when P_j lies between $\widehat{P}_j(V,\lambda)$ and $\overline{P}_j(V)$. Notice that when $\lambda = 0$, i.e. when the entire quota is auctioned off, Case One always applies.

In Case Two, when $\hat{P}_j(V,\lambda) > \overline{P}_j(V)$, it can be shown that Firm i will price along \widetilde{B}_i if $P_j < \overline{P}_j$, and along B_i if $P_j > \hat{P}_j$; however, when $\overline{P}_j < P_j < \hat{P}_j$, Firm i will choose either \widetilde{B}_i or B_i , whichever leads to a bigger profit.⁸

III. OPTIMAL POLICY

In order for the home country to obtain some license revenue, it has to retain some of the import licenses, i.e., it has to set $\lambda < 1$. The symmetric Nash equilibrium price charged by both firms is given by the intersection of \widetilde{B}_1 and \widetilde{B}_2 if the license price is positive in equilibrium:

$$\widetilde{P}(V,\lambda) = \frac{V}{2} \left(\frac{2-\lambda}{k+1} \right). \tag{13}$$

This gives:

$$\widetilde{L}(V,\lambda) = \frac{V[\lambda(k-1) - 3k + 1] + 2A(k+1)}{2(k^2 - 1)}$$
(14)

where $\widetilde{L}(V,\lambda)$ is obtained by setting $P_1=P_2=\widetilde{P}(V,\lambda)$ in Equation (4). Figure 1 shows the combinations of V and λ where the equilibrium license price is positive, i.e., points to the left of the $\widetilde{L}(V,\lambda)=0$ line. The equilibrium license price as a function of V and λ can therefore be characterized in the following way:

$$L(V,\lambda) = \begin{cases} \widetilde{L}(V,\lambda) & \text{if } 0 \le V \le \overline{V} \text{, for any } \lambda \\ \widetilde{L}(V,\lambda) & \text{if } \overline{V} \le V \le V^F, \ \overline{\lambda}(V) \le \lambda \le 1 \\ 0 & \text{if } \overline{V} \le V \le V^F, \ 0 \le \lambda \le \overline{\lambda}(V) \end{cases}$$
(15)

⁷ The proof is shown in the appendix.

⁸ The proof is shown in the appendix.

where:

$$\overline{V} = \frac{2A(k+1)}{3k-1}$$

$$\overline{\lambda}(V) = \frac{1}{k-1} \left(3k - 1 - \frac{2A(k+1)}{V} \right)$$
(16)

from (14). \overline{V} is the quota level that results in a zero license price when no licenses are awarded to the firms ($\lambda = 0$); $\overline{\lambda}(V)$ is the fraction of licenses awarded to the firms (given a quota level, V) that results in a zero license price.

The welfare function is characterized as follows:

$$W(V,\lambda) = \begin{cases} CS(V) + (1-\lambda)V\widetilde{L}(V,\lambda) & \text{if } 0 \le V \le \overline{V}, \ 0 \le \lambda < 1 \\ CS(V) + (1-\lambda)V\widetilde{L}(V,\lambda) & \text{if } \overline{V} \le V \le V^F, \ \overline{\lambda}(V) \le \lambda < 1 \\ CS(V) & \text{if } \overline{V} \le V \le V^F, \ 0 \le \lambda \le \overline{\lambda}(V) \end{cases}$$

$$CS(V) \qquad \text{if } \lambda = 1, \text{ for any } V$$

$$(17)$$

where CS(V) denotes consumer surplus associated with a quota of size V. Consumer surplus can be calculated using the $pari\ passu$ demand function, which results when both firms move their prices together. This is given by Q = A - (k-1)P for each firm, and is illustrated in Figure 2 as a downward sloping line. Free trade welfare is thus two times the area EDP^F . The upward sloping line represents Equation (13). Each firm will price along $\widetilde{P}(V,\lambda)$ as long as the license price is positive, and charge P^F when the license price is zero. The line $\widetilde{P}(V,\lambda)$ swivels downward as λ increases from 0 to 1. For example, if $\lambda=0$, then for a quota $V<\overline{V}$, the price charged by each duopolist is \widetilde{P}^V but the consumer pays P^V , so the license price is $(P^V-\widetilde{P}^V)$. Home welfare is thus twice the area $EBC\widetilde{P}^V$. Note that this cannot exceed the size of triangle EDP^F , hence when $\lambda=0$, any quota below the free trade level V^F is necessarily welfare worsening compared with free trade.

The welfare function $W(V, \lambda)$ is a well-behaved, concave function with respect to λ for any V, attaining its maximum at $\lambda(V)$, where:

$$\lambda(V) = \left(\frac{2k-1}{k-1}\right) - \frac{A}{V} \left(\frac{k+1}{k-1}\right). \tag{18}$$

Since λ is a non-negative fraction, it is defined to be zero for quotas smaller than $V^* = A(k+1)/(2k-1)$.

When (18) is substituted into $W(V, \lambda)$, the resulting function, $W(V, \lambda(V))$, is concave for $0 \le V \le V^*$ and convex for $V^* \le V \le V^F$:

$$W(V,\lambda(V)) = \begin{cases} \frac{V^2}{4(k-1)} + V\left(\frac{2A(k+1) - V(3k-1)}{2(k^2 - 1)}\right) & \text{if } 0 \le V \le V^* \\ \frac{V^2}{4(k-1)} + \frac{[A(k+1) - kV]^2}{2(k-1)^2(k+1)} & \text{if } V^* \le V \le V^F. \end{cases}$$
(19)

For a quota smaller than V^* , the optimal λ is zero, and as demonstrated in Figure 2, $\lambda=0$ always results in a welfare level inferior to the free trade welfare. However, for V between V^* and V^F , it is possible to find an optimal fraction of licenses to give away which would result in the firms charging a price below the free trade price, P^F . Since $W(V,\lambda(V))$ is convex for $V^* \leq V \leq V^F$, a corner solution applies. Clearly, the home government can always do at least as well as free trade by setting $V = V^F$ and varying the fraction of licenses it gives to the duopolists. Consumer surplus associated with a quota of V^F is exactly the free trade welfare level. As long as λ exceeds $\overline{\lambda}(V^F)$, the licenses will have positive value in equilibrium, and as long as λ is not equal to one, the license revenue adds to home welfare. When the quota is set at its maximum level, i.e. the free trade level V^F , the optimal choice of λ is given by Equation (18) as:

$$\lambda^* = 1 - \frac{1}{2k}.\tag{20}$$

The price charged by the duopolists will then be:

$$P^* = \frac{V^F}{2} \left(\frac{2 - \lambda^*}{k + 1} \right) \tag{21}$$

where $P^* < P^F$. The welfare level is:

$$W^* = CS(V^F) + (1 - \lambda^*)V^F L(V^F, \lambda^*) > W^F.$$
(22)

Therefore, the optimal policy for the importing country is to set the quota at the free trade level, V^F and give $\lambda^* V^F/2$ licenses to each duopolist. Notice that λ^* is positively related to the parameter k. In the limit when $k \to 1$, $\lambda \to 1/2$. In this model, therefore, the home country will always choose to give away at least half of the licenses. As $k \to \infty$, $\lambda \to 1$. The larger k is, the more market power each duopolist has, since the two products are less perfect substitutes. As a result, they have to be tempted with a larger share of the license revenues before they will be willing to lower their prices.

IV. CONCLUSION

This paper adds to the literature on auction quotas with imperfect competition by analyzing the outcome of a scheme—first proposed by Krishna (1991)—whereby the importing country gives a fraction of the quota licenses to the foreign suppliers and auctions off the rest. In the example considered here—with the restricted good supplied by Bertrand duopolists facing linear demand functions—the importing country can make itself better off than it was under free trade by setting a quota at the free trade level and sharing the quota licenses with its foreign suppliers. This is true even when the home market is perfectly segmented from the rest of the world. By giving some of the licenses to the foreign duopolists, the importing country creates an additional incentive for the firms to keep their prices down so as to maintain a positive license price. At the same time, by retaining some of the licenses for itself, it is able to reap some of this positive license revenue.

Figure 1. (V, λ) Combinations Resulting in a Positive License Price

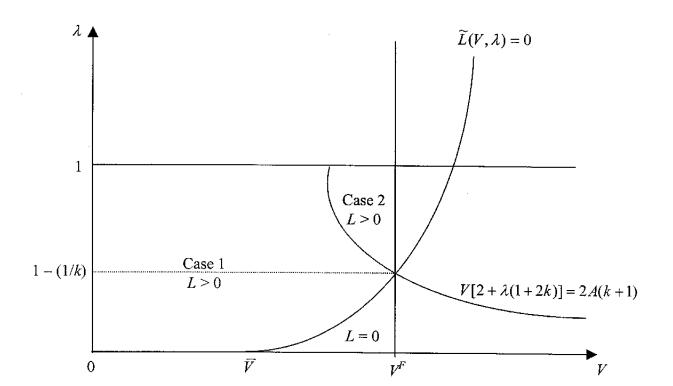
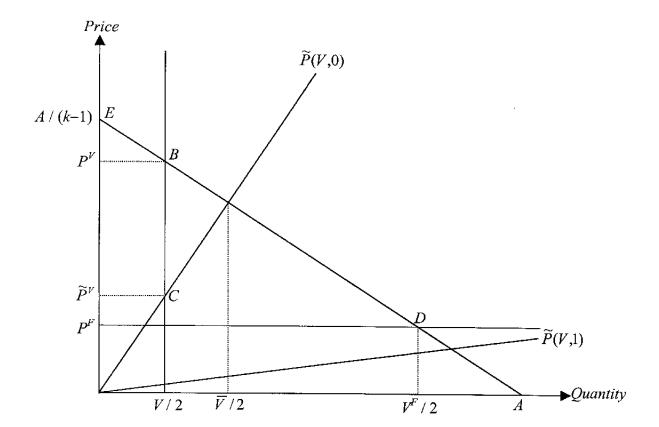


Figure 2. Pari Passu Demand Function



DERIVATIONS OF CASE ONE AND CASE TWO

A. Case One: $\hat{P}_2(V,\lambda) < \overline{P}_2(V)$

Define P_1^0 as the price set by Firm 1 which satisfies $L(\cdot) > 0$ for any given P_2 . From (8), $P_1^0 = [(2-A)/(k-1)] - P_2$. Recall that $\widetilde{\pi}_1$ is Firm 1's profit when the license price is positive (i.e., when $P_1 < P_1^0$) and π_1 is its profit when $P_1 > P_1^0$. Firm 1's profit functions are illustrated in Figures A.1, A.2, and A.3 for the case where $\partial \pi_1 / \partial P_1 < \partial \widetilde{\pi}_1 / \partial P_1$ and $\partial \pi_1 (\cdot) < \widetilde{B}_1 (\cdot)$, although the following reasoning holds as well if $\partial \pi_1 / \partial P_1 > \partial \widetilde{\pi}_1 / \partial P_1$ and/or $\partial \pi_1 (\cdot) > \widetilde{B}_1 (\cdot)$.

When $P_2>\overline{P}_2$, $\partial\pi_1/\partial P_1>0$ and $\partial\widetilde{\pi}_1/\partial P_1>0$, i.e., both π_1 and $\widetilde{\pi}_1$ are increasing in P_1 —in Figure A.1, both $B_1(\cdot)$ and $\widetilde{B}_1(\cdot)$ lie to the right of P_1^0 , and since π_1 is Firm 1's profit when $P_1>P_1^0$, Firm 1 maximizes profit by charging $B_1(\cdot)$. When $P_2<\hat{P}_2$, both π_1 and $\widetilde{\pi}_1$ are falling in P_1 —in in Figure A.2, both $B_1(\cdot)$ and $\widetilde{B}_1(\cdot)$ lie to the left of P_1^0 , and since $\widetilde{\pi}_1$ is Firm 1's profit when $P_1<P_1^0$, Firm 1 maximizes profit by charging $\widetilde{B}_1(\cdot)$. Between \widehat{P}_2 and \overline{P}_2 , $\partial\pi_1/\partial P_1<0$ and $\partial\widetilde{\pi}_1/\partial P_1>0$ at P_1^0 —in Figure A.3, Firm 1 maximizes profit by charging $P_1=P_1^0$.

Case One is illustrated in Figure A.4 for $V = V^F$. Note that when $\lambda = 0$ and $V = V^F$, the license price is zero in equilibrium. However, the equilibrium license price is positive for quotas smaller than V^F .

B. Case Two: $\hat{P}_2(V, \lambda) > \overline{P}_2(V)$

Now consider Case Two, where $\hat{P}_2(V,\lambda) > \overline{P}_2(V)$. For values of P_2 exceeding \hat{P}_2 , both π_1 and $\widetilde{\pi}_1$ are increasing in P_1 —as before, this means Firm 1 will maximize profit by charging $B_1(\cdot)$. For values of P_2 below \overline{P}_2 , both π_1 and $\widetilde{\pi}_1$ are falling in P_1 —again, as before, this means Firm 1 will maximize profit by charging $\widetilde{B}_1(\cdot)$. However, when P_2 lies between \overline{P}_2 and \hat{P}_2 , $\partial \pi_1 / \partial P_1 < 0$ and $\partial \widetilde{\pi}_1 / \partial P_1 > 0$ at P_1^0 . The profit functions in this case are illustrated in Figure A.5—Firm 1 will choose either $B_1(\cdot)$ or $\widetilde{B}_1(\cdot)$, whichever leads to a bigger profit.

Case Two is illustrated in Figure A.6 for $V = V^F$ and $\lambda = 1$. This corresponds to a VER at the free trade quantity, with all the quota licenses given away to the foreign producers. Between $\overline{P_2}$ and $\hat{P_2}$, P_1 jumps around between $B_1(\cdot)$ and $\widetilde{B_1}(\cdot)$ so Firm 1's reaction function is

not drawn in that region. The same applies to Firm 2's reaction function in the region between \overline{P}_1 and \hat{P}_1 . In this case, there can be any number of mixed strategy equilibria but only one pure strategy equilibrium at the intersection of $B_1(\cdot)$ and $\widetilde{B}_1(\cdot)$ with a positive equilibrium license price. However, since $\lambda=1$ in this example, all the license revenue accrues to the exporters.

Figure A.1. Firm 1's Profit Functions when $P_2 > \overline{P_2}$

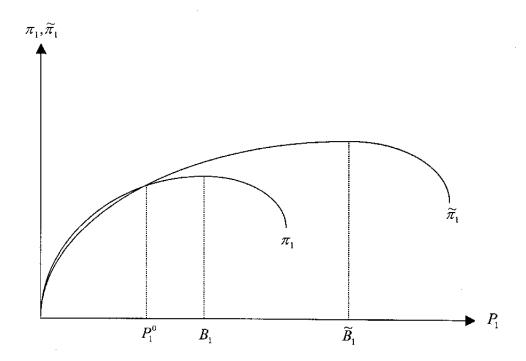


Figure A.2. Firm 1's Profit Functions when $P_2 < \hat{P}_2$

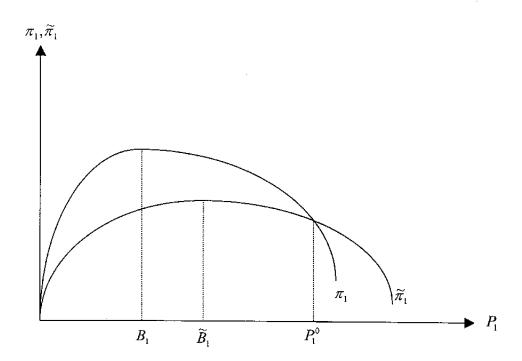


Figure A.3. Firm 1's Profit Functions when $\hat{P}_2 < P_2 < \overline{P}_2$

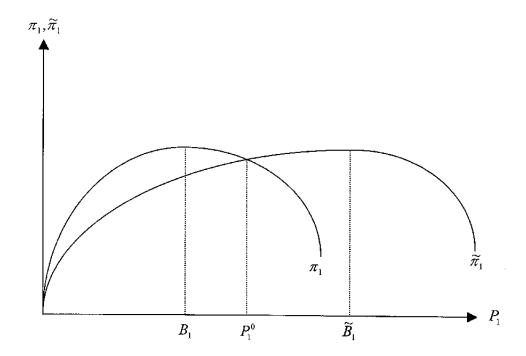


Figure A.4. Case One: Quota at the Free Trade Quantity

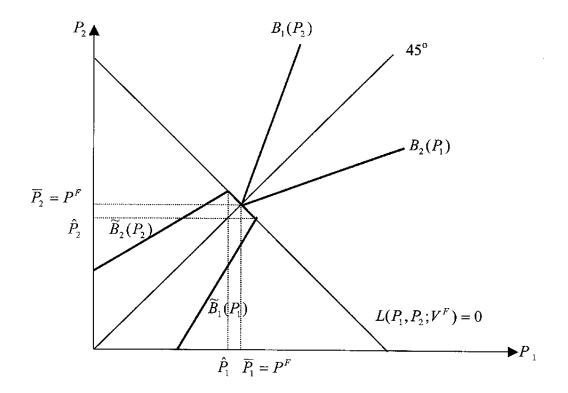


Figure A.5. Firm 1's Profit Functions when $\overline{P}_2 < P_2 < \hat{P}_2$

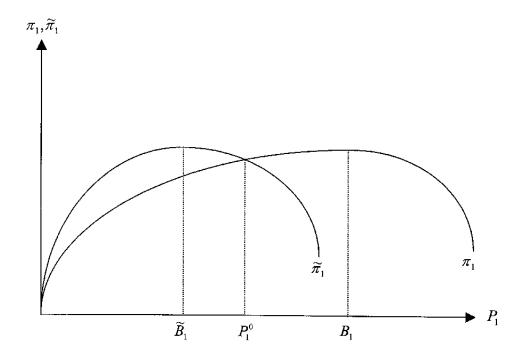
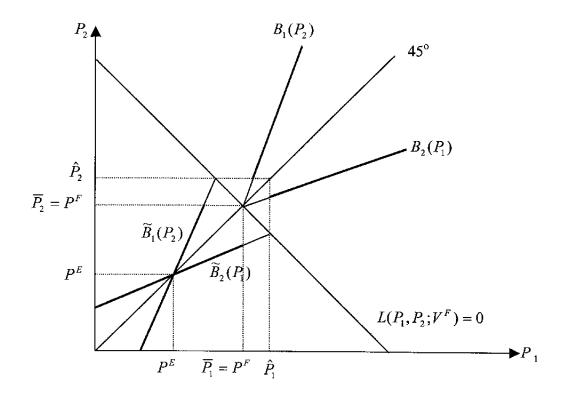


Figure A.6. Case Two: VER at the Free Trade Quantity



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